## Verification Example 2 for Heat Module

This verification example compares the MODFLOW-USG solution with the Stallman (1965) analytical solution for transient heat flow in the subsurface in response to a sinusoidally varying temperature boundary at land surface. The problem consists of a saturated vertical soil column 60 m long having a hydraulic conductivity of 1.0 x 10-4 m/sec (28.35 ft/d). Downward groundwater flow with a Darcy velocity is of 5 x 10-7 m/sec is generated by a constant head of 60 m at the top and 59.7043 m at the bottom. With a porosity of 0.35, the pore velocity is 12.3 cm/d = 1.42 x x 10-6 m/sec. The column is discretized int0 200 cells with a uniform thickness of 0.5 m each.

The transport model is simulated with both heat and equivalent solute conditions. The initial temperature of the subsurface is 10 0C, and the temperature at the surface varies as Ttop = 10+5sin(2pi t/T) where t is the time and T is the wave length of 1 year. The heat transport parameters and equivalent solute parameters are shown on Table H1 below.

***Table H1: Heat and equivalent solute transport parameters for variation example of heat module***

|  |  |  |  |
| --- | --- | --- | --- |
| Symbol | Variable | Value | Unit |
| n | Porosity | 0.35 | (–) |
| λw | Effective thermal conductivity of water | 0.58 | (W/m/K) |
| λs | Effective thermal conductivity of soil | 2 | (W/m/K) |
| λm | Effective thermal conductivity of porous medium | 1.503 | (W/m/K) |
| ρw | density of water | 1000 | (kg/m3) |
| cw | Specific heat capacity of water | 4.17E+03 | (J/kg/K) |
| ρs | Density of solid material | 2650 | (kg/m3) |
| cs | Specific heat capacity of solid | 800 | (J/kg/K) |
| ρb | Dry bulk density | 1709.5 | (kg/m3) |
| csv | volumetric heat capacity of solid | 2.12E+06 | (J/m3/K) |
| Kd | Partition coefficient | 1.92E-04 | (m3/kg) |
| R | Retardation factor | 1.94 |  |
| αl | Longitudinal dispersivity | 0 | (m) |
| αt | Transverse dispersivity | 0 | (m) |
| Do | Thermal diffusivity (diffusion coefficient) | 1.03E-06 | (m2/s) |
| key | required for heat equation solution |  |  |
| key | required for equivalent solute equation solution |  |  |
| key | Intermediate calculation |  |  |
| key | required for both solutions |  |  |

Results of the simulation at 10 years are shown on Figure H1. The Stallman analytical solution and MODFLOW-USG heat and equivalent solute transport simulations compare very well.

**Figure H1:** Comparison of temperature profile simulated with MODFLOW-USG as heat or equivalent solute transport against the Stallman (1965) analytical solution at 10 years.